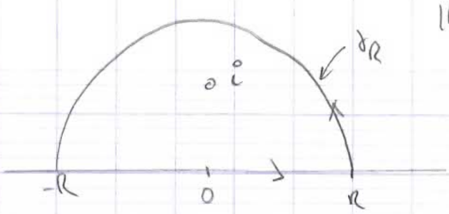

$|R|>1$

On a $J_\alpha = H_1(R) + H_2(R)$

$$H_1(R) = \int_{-R}^{R} \frac{x^\alpha}{1+x^2}\,dx$$

$$H_2(R) = \int_0^{\pi} \frac{e^{\alpha \ln(R) + i\theta}}{1+R^2 e^{2i\theta}}\, e^{i\theta} i R\, d\theta$$

$$|H_2(R)| \leq \int_0^{\pi} \frac{e^{\alpha \ln(R)}}{R^2 - 1} R\, d\theta = \pi \frac{R e^{\alpha \ln(R)}}{R^2 - 1} \qquad e^{\alpha \ln(R)} = R^\alpha$$

$$\leq \pi \frac{R^{1+\alpha}}{R^2-1} \underset{R\to\infty}{\sim} \pi R^{\alpha-1} \xrightarrow[R\to\infty]{} 0 \qquad (\alpha<1)$$

Par le théorème des résidus, on a :

$$J_\alpha = 2\pi i \operatorname{Res}(f, i) = 2\pi i \lim_{z\to i} \frac{z^\alpha (z-i)}{(z+i)(z-i)} = 2\pi i \frac{i^\alpha}{2i} = \pi i^\alpha$$

$$i^\alpha = e^{\alpha i \frac{\pi}{2}}$$